A Space Odyssey:
Mathematical journeys across uncharted territory

Inaugural Lecture of

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Professing..... Space Geodesy
Space Geodesy Science and Engineering: the underlying mathematics and models for satellite navigation (GPS)
Space Geodesy Science and Engineering: GPS derived global velocity vectors
Space Geodesy science and engineering:
Sea level variations from Space

Jason-1 satellite sea level model from one 10 day pass
Red = higher than normal sea level (≈20cm)
Space Geodesy and Society: El Nino, the Dust Bowl and the Great Depression
Project: Low Earth Orbiter (LEO) attitude determination
Low Earth Orbiter (LEO) attitude determination: the maths

\[\begin{align*}
\left\{ \begin{array}{l}
\text{LLS}_i - n_{11}x_A^{BFS} + m_{12}y_A^{BFS} + m_{13}z_A^{BFS} + \sqrt{\lambda} + \text{LLS}_j - n_{21}x_A^{BFS} + m_{22}y_A^{BFS} + m_{23}z_A^{BFS} \\
\text{LLS}_i - n_{31}x_A^{BFS} + m_{32}y_A^{BFS} + m_{33}z_A^{BFS}
\end{array} \right\}^{1/2} \\
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\end{array} \right\}^{1/2}
\end{align*}\]

\[\text{N}^{ij}_{AB} + \lambda \Phi^{i}_{A} - \Phi^{i}_{B} - \Phi^{j}_{A} + \Phi^{j}_{B} \]

\[= \lambda \left( \Phi^{i}_{A} - \Phi^{i}_{B} - \Phi^{j}_{A} + \Phi^{j}_{B} \right) \]

Publications:


Project: OSGM05 - the UK gravity field model

- Mathematical, gravity-based model used as the Ordnance Survey’s national height datum
- Derived from multiple forms of satellite-derived data products
- Incorporated into tens of thousands of computers, data loggers, science-grade GPS receivers
- On-going developments with OS to release new generation of models

Publications:
Project: Vertical Offshore Reference Frame (VORF)

- Commissioned by UK Hydrographic Office
- Developed mathematical models of sea surfaces
- Operational sea surface models now used routinely in offshore work
- Approach from the Royal Navy to develop a global model

Publications:
Projects:

Orbit determination accuracy: cm to mm
Orbit prediction accuracy: m to cm
Forces acting on a GPS spacecraft

- Antenna thrust (AT)
- Thermal forcing (TRR)
- PRP (planetary radiation pressure)
- Solar gravity
- Solar radiation pressure (SRP)
- Lunar gravity
- Tidal effects
- Earth gravity
- Planetary gravity
- General relativistic effects
Solar radiation pressure

Thermal Re-radiation forces

How big are these forces? What effect do they have?

Resultant force from Thermal emissions

Resultant force from solar photons
Photon force modelling

Pixel array simulating photon flux

Computer model of spacecraft
radiation flux

regions of shadow computed automatically

Secondary intersections
Pixel array rotated around spacecraft, solar panels adjusted

Publications:
NASA Jet Propulsion Laboratory, Pasadena, California
Earth radiation flux incident on a GPS satellite
Force modelling on GPS satellites

- Computer codes adapted to run on supercomputers
- Contract research for US Air Force/JPL
- Won several international awards
- Orbit determination accuracy of 2cm achieved
- Earth radiation pressure modelling adopted as international standard (IGS)
- Commercial contract to develop orbit prediction and orbit determination code for one of the world’s largest providers of GPS positioning services
Along-track orbit prediction errors over 12 hours for one GPS satellite with different photon-based force models:

- **Solar radiation pressure (SRP) only**
- **SRP + Thermal force (TRR)**
- **SRP + TRR + Antenna Thrust (AT)**
- **SRP + TRR + AT + Planetary Radiation**

Orbit Prediction Error (m) vs Time (hours) with 8 metre orbit error.
Jason-1 modelling (SRP and TRR)

- Extensive tests carried out at JPL
- Dynamic orbit improvements in cross overs, SLR residuals, orbit overlaps and scale factors
- Model subsequently tested by Goddard Space Flight Centre
- North hemisphere – South hemisphere mass transfer anomaly resolved
- Anomalous 60 day period signature removed
- Model adopted by NASA as operational standard
Summary of orbit modelling research

Publications:
Designing a navigation system for manned and robotic missions to Mars
European Space Agency project

- Why build a navigation system?
- Design the space segment
  (spacecraft instruments, orbit configuration)
- Design the surface-based system components
- Simulate and test system performance
  at three landing sites
Space segment design

- **Inclined orbit**
  - Sinus Meridiani
  - Eos Chasma
  - Holden Crater

- **Polar orbit**
  - Sinus Meridiani
  - Eos Chasma
  - Holden Crater

- **Equatorial orbit**
  - Sinus Meridiani
  - Eos Chasma
  - Holden Crater

- **Satellite ground track**
- **5° mask angle zone**
- **Beacon location**
Holden Crater Mission Traverses

- Missions using pressurised rover vehicles with two astronauts
- All missions are ‘over the horizon’
- Each traverse may take several days
Sample simulation results: four beacons, one way ranging, clock resets on orbiter pass.
The geometry of Roman amphitheatres

Sarmizegetuza, Romania  Guildhall amphitheatre, London

Publications:

National Centre for Earth Observation: Dynamic Earth and Hazards

- New instrument design
- Seismometry/high rate multi-constellation GNSS
- Sidereal filter
- Real-time tsunami and earthquake warning system
- Aim: order of magnitude more sensitive than current instruments
- Funding: ~62 PhDs
- Value: ~£11M
- Inspired by: Flooding; Foot and Mouth outbreak; Jared Diamond’s ‘Collapse’
- Funding from EPSRC and industry
What is the future of Space Geodesy?

Has it even arrived yet?

Why is there a picture of Dr Who on the screen?
Vice Dean for Research
FACULTY OF ENGINEERING SCIENCES

- Biochemical Engineering
- Chemical Engineering
- Civil, Environmental and Geomatic Engineering
- Computer Science
- Electrical and Electronic Engineering
- Jill Dando Institute for Crime Science
- Management Science and Innovation
- Mechanical Engineering
- Medical Physics and Bioengineering
- School of Energy and Resources, Australia
Looking forward:

Governing Board Member
International GNSS Service (IGS)
Working Group Leader:
Orbit Dynamics and Space Vehicle Attitude
Looking forward:

Space Geodesy and Navigation Laboratory
Looking forward:

Space Geodesy: challenges and opportunities

- Developing effective early warning systems for natural hazards
- Understanding the exchange mechanisms between the cryosphere, atmosphere, ocean and solid earth
- Modelling and predicting how sea level and the ice caps will respond to climate change
- 120 ‘science grade’ navigation satellites on orbit
- Ultra stable spatial and time reference frames
- Public understanding of the value of space-based measurement platforms
- A generation of scientists who have grown up with space technology
Personal Research Themes (technical)

- Develop understanding and application of orbit dynamics modelling at higher altitudes (GEO to interplanetary)
- Extend orbit dynamics techniques at LEO altitudes (ram profiling, surface charge coupling with terrestrial magnetic field, rigorous earth radiation effects)
- Extend modelling techniques to space debris constellation prediction
- Build on understanding of atomic clocks/timescale realisation (with National Physical Laboratories), linking to orbit dynamics
- Roll out our research in orbit dynamics/clocks to UK industry (chip manufacturers)
- Maintain world lead in orbit dynamics
- Develop training for PhD/Post-Doctoral Researchers
- Build on the group’s expertise and collaborations with UK and international partners
Thanks to:

Partners and clients to date...
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Conclusions
Thanks for listening
Please stay tuned
….and watch this space